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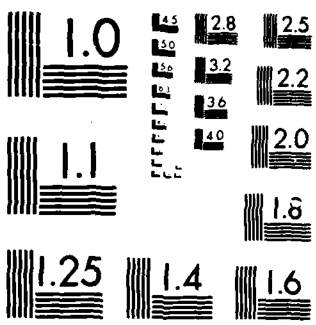

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NCEL
Contract Report

An Investigation Conducted By R.L. Taylor

Sponsored By Naval Facilities Engineering Command

METHODS FOR SOLUTION OF DYNAMIC CONTACT PROBLEMS

AD-A163 553

ABSTRACT Methods for treating dynamic soil-structure interaction in large, three-dimensional, nonlinear finite element settings are first compared. The report focuses on constraint-type methods in addressing the spatial aspects of the problem of implementing dynamic contact behavior into large-scale finite element calculations for civil-structural systems.

NAVAL CIVIL ENGINEERING LABORATORY PORT HUENEME CALIFORNIA 93043

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METRIC CONVERSION FACTORS

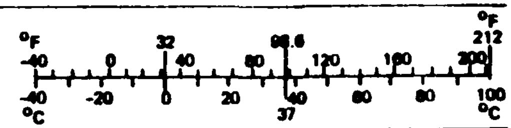
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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tension cut-off capability would seem to most easily satisfy the soil-structure interaction requirement. However, such a formulation does not provide for more general requirements involving contact conditions of structural components. Thus the report focuses on constraint-type methods in addressing the spatial aspects of the problem of implementing dynamic contact behavior into large-scale finite element calculations for civil-structural systems.

Of the constraint-type methods, one in particular known as the augmented Lagrangian formulation, is deemed most appropriate to the specified class of problems. This method is a combination of the Lagrange multiplier and penalty formulations and aims at retaining only the best characteristics of both methods. It employs an augmentation parameter to vary the relative participation of the Lagrange multipliers and the penalty parameter in the enforcement of contact conditions.

Temporal aspects of the problems are discussed. Specifically, the issue is raised of whether or not special handling techniques for temporal integration such as the introduction of impact-release conditions are required.

Limited numerical experiments suggest that very little additional computational effort is required to obtain the full accuracy of the Lagrange multiplier method. Further, the workable range of the penalty parameter is shown to be extended compared with the conventional method.

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Methods for Solution of Dynamic Contact Problems

by

Robert L. Taylor, PhD.

1. INTRODUCTION

The solution of contact problems by the finite element method has received considerable attention in recent years [2-5,8,11-15,18,19,24-28,32-37,39]. In general the problem is non-linear and, thus, even for problems composed of linear elastic materials is non-trivial to solve. For transient problems, such as those of interest to NCEL, two aspects of the problem need to be considered: a.) the spatial aspects of solution, and b.) the temporal aspects of solution. For the spatial part of the problem solution schemes have been proposed and used as part of finite element solution algorithms. Some of the methods suggested are use of so called "gap" elements and Lagrange multiplier methods [11], penalty function methods [7,12,13,24], perturbed Lagrangian methods [36], and recently augmented Lagrangian methods [37]. The temporal aspect of the problem has received little attention in the literature. In the work of Hughes, et. al., [11] it was stated that use of "impact-release" conditions are necessary to achieve high accuracy in the numerical results. This is contrasted by the development of Hallquist in which no such conditions are included yet plausible results are achieved [8]. Further evaluation is necessary before such results may be widely applied.

In the present study the state-of-the-art for solving large

dynamic contact problems modeled by the finite element method has been reviewed. The problems of interest to NCEL involve such applications as large dry-dock facilities which may experience strong seismic motions, as well as possible accidental blast loading on facilities. Such problems are characterized by the presence of materials which exhibit very strong nonlinear responses (e.g., soils, concrete, etc.). The anticipated deformations for many of the conditions anticipated are expected to remain small so that linear strain measures suffice. The interaction between the "structure" (e.g., a dry dock wall) and the surrounding geological media and the interaction of system elements (e.g., joints, doors, gates, etc.) are expected to involve dynamic contact conditions for the loadings considered. Accordingly, the present study is directed to possible schemes for including dynamic contact modeling, as well as the usual material and geometric nonlinearity, in simulations.

Past studies by NCEL have focused on development of appropriate structural modeling capabilities and constitutive equations for the systems to be considered. Constitutive equations for geologic and concrete materials have been developed and studied [9,10], and structural modeling methods for plate/shell systems have been addressed [16,31]. In addition studies have been conducted by Nour-Omid and Taylor [20,21,22,23,34] on effective methods to solve the large algebraic system of equations resulting in this class of non-linear finite element problems. Finally, Shugar and Cox have addressed issues related to combining finite element and boundary element methods [30]. In future efforts NCEL will be directing

efforts to bring these studies together in a design-oriented software system which will permit engineers to make quantitative assessments of the behavior of the soil-structural systems subjected to transient design loads. This software system will provide information to redesign the structure for loadings which do not achieve specified limits of safety. In order to bring together the previous efforts it is necessary at this time to investigate the most feasible methods for including the contact interaction with the other salient features of the non-linear problems involved.

2. METHODS SURVEYED

In the present study several methods were surveyed as possible candidates for including dynamic contact interaction effects. The methods considered include:

a.) Frequency domain methods,
b.) Use of non-linear material characteristics and no contact models.
c.) Lagrange multiplier, penalty and other associated methods,

Each of these methods is summarized and evaluated below.

2.1 Frequency Domain Methods

In the analysis of seismically loaded soil-structure systems a frequency domain solution is often advocated. For situations involving non-linear material response an equivalent linear material model based upon some mean measures of the response is introduced. Such methods are widely used in the design of nuclear power plants. Recently, Roesset has mentioned the

possibility of including the interaction effects of separating boundaries by an impedance compensated solution [29]. No numerical results have been included to indicate possible results with this method. The primary difficulty with use of frequency domain solutions is the development of effective "mean" properties for each of the model constituents. While some success has been achieved for analysis of soils there is no indication that the method can be applied in general to a wide class of problems. Indeed if the method is to be adopted it will be necessary to qualify the results by an extensive set of experiments or by a refined objective numerical modeling using a more accurate deterministic model. Accordingly, it is recommended that this approach not be followed at this time and that efforts be devoted to the implementation of a solution methodology in the time domain. There are additional reasons to do this. For strong blast loading there may be a zone in which finite deformation effects are required. Also for any situation in which "unstable" mechanisms (e.g., buckling) may develop it also will be necessary to use a geometrically nonlinear formulation. At this time there is no evidence that frequency domain solutions can incorporate both geometric and material nonlinearity in the same solution procedure. It is believed that considerable research would be involved to even assess the feasibility of this approach. Finally, the frequency domain approach often does not result in significant savings in compute times. On the other hand there is an extensive literature on formulation and solution of geometrically and materially nonlinear problems in the time domain.

2.2 Use of Nonlinear Material Models for Interaction Simulation.

Several non-linear finite element computer programs have already been written to solve specific problems which involve both geometric and material nonlinearity. The most notable programs are probably ABACUS, ADINA, MSC/NASTRAN and the DYNA and NIKE programs developed by Hallquist at LLNL. It should be noted that none of these programs is a design oriented system such as that proposed by NCEL. Many of these nonlinear finite element programs include material models which incorporate a "tension cut-off" on stresses or deformations. In modeling granular materials such as soils the inclusion of a tension limiting constitutive assumption is essential for proper modeling of the material behavior. In modeling soil-structure interactions the limiting tension will effectively provide a release mechanism of the structure from the soil media. In this class of problems there may be little need to incorporate a general contact condition in the modeling of the system since the tension limiting aspect of the soil constitutive equation will essentially eliminate boundary interaction effects. On the otherhand, for interactions between structural elements there is still a need to model the interaction effects as reflected by a general finite element contact simulation.

2.3 Lagrange Multiplier and Penalty Methods.

A methodology to handle dynamic contact simulations is included in both DYNA (an explicit method finite element program) and NIKE (an implicit method finite element program). These

programs are widely used at LLNL with considerable success in modeling dynamic contact interactions. As noted previously, neither of these programs include any special treatment for "contact-release" conditions.

The modeling for the spatial treatment of contact problems may be incorporated in a finite element program by introducing into the weak form of the balance of momentum a Lagrange multiplier constraint on the "gap". By monitoring the "gap" between two surfaces the Lagrange multiplier can be controlled to introduce the correct interaction forces whenever a contact condition is detected (e.g., see [11]). In a discrete form of the condition it is necessary to describe the "gap" in some consistent way. Various approaches have been proposed to handle this aspect. In the earliest works deformations were considered as small and it was assumed that nodes on one surface would "contact" at nodes on the other surface (node-on-node contact). This leads to a very simple form of the Lagrange multiplier method to describe the "gap". Selecting a sign convention to which a positive gap corresponds to no interaction, a negative gap implies penetration and it is merely necessary to introduce the Lagrange multiplier constraint to force the gap to a "zero" value. The interaction forces are "nodal" in nature and may be monitored to ensure that tension does not exist on the surface. Accordingly, the form of the Lagrange multiplier to be added to the weak form may be deduced by an appropriate linearization of the term

$$- F\ (x_1 - x_2) \tag{1}$$

where F is the nodal interaction force (Lagrange multiplier), and

x_1 and x_2 are the deformed positions of a "node" for surfaces 1 and 2. This approach is extremely simple to program and may be included as an "element" subprogram in any nonlinear finite element program (e.g., see Appendix A where such a routine is given for the FEAP program developed by the author [31]). While this approach is very simple to program it is not sufficiently general to permit the solution of realistic contact problems. Indeed any problem involving rolling, sliding, or large deformations will invariably lead to nodes on one surface not contacting nodes on the other surface. In addition modeling of a structure for dynamic analysis may require different mesh spacing for each component or different material.

It is relatively straight forward to extend the nodal Lagrange multiplier method to a method where a node on one surface interacts with a general (non-nodal) point on the other surface. For two-dimensional problems a Lagrange multiplier term may be introduced in the form

$$F \left(x_1 - (1-a)\, x_{2i} - a\, x_{2j} \right) \tag{2}$$

where "i" and "j" define two nodes on the 2-surface being contacted by the node on the 1-surface and "a" defines the position parameter of the contact point. The algorithm is complicated now by the need to develop search methods to find the nodal pair "i" and "j" where contact occurs and to determine the value of "a".

The method described in the previous paragraph is quite general; however, some deficiencies have been noted in its application. The first problem is that there is a bias to the

treatment of the surfaces. Only nodes on one surface are in contact with the other surface. The nodes of the other surface may not be in contact or may penetrate. Indeed the second deficiency is this latter point. If several nodes exist on surface "2" between adjacent contacting nodes on surface "1" they may penetrate the "contacting" surface in an uncontrolled manner (see Figure 1.). This is most undesirable for dynamic problems where inertial effects will not be correctly modeled near the contacting surfaces. Hallquist proposed to treat the problem by a two pass approach. In the second pass the role of bodies "1" and "2" are interchanged thus averaging the contact conditions. For sufficiently fine meshing this is an effective approach. However, as shown by Wriggers, et. al., for coarse mesh regions the results may not be esthetically pleasing [37]. Wriggers proposes using the average gap on segments instead of nodal values to define the contact conditions. While this leads to more reasonable pictorial forms of the contact it is more difficult to determine all permutations for the possible segments, especially for 3-dimensional situations. Some additional effort is needed to define appropriate discrete forms of the gap for 3-dimensional applications.

An additional complication from the introduction of a Lagrange multiplier treatment of the contact condition is that the linearized equations of the finite element model are indefinite. Indeed in a treatment by Newton methods with a direct solution of the linear algebraic equations, zero diagonals occur for each interaction force degree-of-freedom. For static problems direct solution procedures without pivots can fail. It

has been traditional to not use pivots when solving the large linear algebraic equations, hence this aspect will be troublesome for programs using direct solution procedures. Often the problem may be avoided by a judicious numbering of the degrees-of-freedom in the global equations. Since many programs use automatic renumbering algorithms it will be necessary to introduce additional checks to avoid producing a zero pivot in the renumbered equations. A second problem is that additional degrees of freedom are introduced into the global equations to account for the interaction effects. Thus even if direct solutions are replaced by appropriate iterative procedures (e.g., the Newton-Lanczos method recommended and studied previously for NCEL [20,34]) these additional equations must be retained. Finally, checking convergence requires separate treatments for the force and displacement parameters to properly monitor performance. It is possible to eliminate the interaction forces "F" from the global equation set by introducing a penalty form for the Lagrange multiplier term. This is very effective, and furthermore removes the indefinite character of the resulting linearized form for static applications of stable material models. The primary difficulty with the penalty method is the need to select an appropriate "penalty parameter". If the parameter is too small undesirable penetrations will result, whereas, if the parameter is too large numerical ill conditioning problems result. Felippa [7,8] has published results which illustrate the difficulties which may result from using a penalty approach. The difficulties he cites have been encountered often

by us in using penalty methods for the contact problem as well as in the enforcement of near incompressibilty conditions in finite deformation elasticity and plasticity problems solved by the finite element method. Precise guidelines cannot as yet be established to indicate when the selection of the penalty parameter will lead to numerical ill-conditioning or a decrease in rate of convergence of Newton's method. Accordingly, it is a problem which must be addressed further before large finite element programs can be developed for general application. As a pilot study an effort was initiated to evaluate the effectiveness of using an augmented Lagrangian approach in conjunction with a penalty form of the contact constraint.

3. AUGMENTED LAGRANGIAN FORMULATIONS

In an attempt to avoid the numerical ill conditioning problem, the Lagrange multiplier method may be augmented by a penalty term. This results in a problem formulation which combines the best features of both methods. The global equations are normally treated using the penalty term in the linearized coefficient matrix; the Lagrange multiplier term is included as an iterative correction to the right hand side of the equations. The method is generally far less sensitive to the penalty parameter selected. In fact the penalty parameter may be selected as a small number and iterative improvements are used to obtain high precision satisfaction of the constraint term. The number of iterative corrections required to achieve convergence to a specified tolerance is affected by the size of the penalty parameter selected. Thus, the algorithm when properly

implemented requires a penalty parameter of sufficient size to limit the iterations to a reasonable number. The proper value for the penalty parameter in an augmented Lagrangian form is much smaller than that required for the penalty term alone. For example, a problem solved by the penalty method and requiring a penalty parameter of 10^6 to achieve "satisfaction" of the constraint may be solved using a value of 10^2 to 10^3 for the augmented method which also will achieve better accuracy with little increased effort.

To illustrate a use of the augmented Lagrangian method let us consider a node-on-node contact formulation as defined by Eq. (1). The term is augmented by the penalty term as given by Eq. 3.

$$(3) \qquad - F (x_1 - x_2) + 0.5 (1 - s)/E (F)^2$$

The penalty parameter is given by "E" and "s" is a parameter which when zero defines the penalty method and when unity gives the Lagrange multiplier method. Linearization of this term leads to the form

$$(4) \qquad \begin{vmatrix} 0.0 & 0.0 & -1.0 \\ 0.0 & 0.0 & 1.0 \\ -1.0 & 1.0 & 1/E \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ F \end{vmatrix} \quad - s/E (F)$$

Elimination of F in the first two equations by using the last equation gives the penalty method augmented by the "s" term as:

$$(5) \qquad \begin{vmatrix} E & -E \\ -E & E \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} - s F \\ s F \end{vmatrix}$$

Felippa has discussed several possible iterative methods to incorporate the Lagrange multiplier term in an iterative manner.

Of those considered the hybrid method which is identical to an augmented Lagrange method proposed in [37] is recommended. Other methods are also given in [1,17]. If we define the gap by

(6) $$g(x) = x_1 - x_2$$

and an iterative increment to the position by

(7) $$x^{(i+1)} = x^{(i)} + u^{(i)}$$

for each surface then the Newton equations to be solved at each iteration may be written as

(8) $$\begin{vmatrix} (K_1 + E) & -E \\ & \\ -E & (K_2 + E) \end{vmatrix} \begin{vmatrix} u_1 \\ \\ u_2 \end{vmatrix}^{(i)} = \begin{vmatrix} f_1 - g - F_{(i)} \\ \\ f_2 + g + F_{(i)} \end{vmatrix}$$

where K_1, K_2 are the tangent stiffness quantities and f_1, f_2 are the force terms for bodies 1 and 2, respectively; and "i" is the iteration number. As suggested in [37] the force F may be updated using

(9) $$F^{(i+1)} = F^{(i)} - \min\ (\ E\ g(x)\ ,\ F^{(i)}\ )$$

As the method converges, g(x) tends to zero while F approaches the solution of the Lagrange multiplier method. A generalization of this algorithm for general two-dimensional contact situations has been incorporated into the finite element program FEAP using the method outlined in [37] and sample problems have been solved. Results are summarized below.

4. NUMERICAL EXAMPLES

To assess the performance of the augmented Lagrangian form of the penalty method a series of simple test problems has been analyzed. The only m^thod assessed under the current effort is the perturbed Lagrangian form of the problem. In our implementa-

tion it is not possible to consider an arbitrary meshing of the problem; it is necessary to have only one node between any segment in the model. This is not a limitation of the method, but merely the implementation constructed to date. As will be seen in the recommendations section we propose to use the two pass penalty method for the three-dimensional applications. A second reason for using the two-pass algorithm involves performance of the perturbed method for node-on-node situations (or those which are node-on node to a numerical tolerance) where the algorithm has some aspects of ill-conditioning in defining the segment. The perturbed Lagrangian does have an advantage as will be illustrated in one of the examples considered.

4.1 Two Spring Constraint Problem.

The first example considered is a two spring model for which a specified gap is to be maintained using the augmented penalty method described above. The model and the parameters considered are shown in Figure 2. It should be noted that the "penalty parameter will be placed in parallel with the spring "k2" to maintain the gap distance. A range of values for the penalty parameter was considered with the results obtained plotted in Figure 3. The plateau shown in Figure 4 denotes maximum number of significant digits specified for defining the gap and is attained in very few iterations (e.g., 1-3 iterations for a wide range of penalty values). This example illustrates that the augmented method requires very little additional effort to obtain the full accuracy of the Lagrange multiplier method.

4.2 "Rigid" Punch into an Elastic Foundation.

The second example considers a stiff punch pressed into a flexible elastic foundation; the meshes are shown in Figure 4. This example has been considered also in [37] for the one- and two-pass penalty algorithms and the perturbed Lagrangian algorithm. In this example the modulus of the punch is set two orders of magnitude larger than the elastic foundation to represent the "rigid" condition. The penalty parameter is allowed to vary over a wide range of values ranging from 10^0 to 10^{20}. The accuracy attained (one iteration after the contact region is determined) for the number of significant digits defining the gap is shown in Figure 5. The maximum accuracy specified is 8-digits and is obtained for the augmented method for all values of the penalty parameter between 10^2 and 10^{16}, whereas both the perturbed and standard (one and two-pass) methods attain this accuracy for a range of parameters between about 10^{10} and 10^{14}. The converged solution accuracy is indicated in Figure 6. This example clearly indicates the significant advantages of the augmented approach over a conventional penalty method.

4.3 Dynamic Contact of an Elastic Block on a Rigid Surface

The next example considered is an elastic block impacting a rigid surface. This is a dynamic problem which has a wave like solution involving a situation in which any need for impact-release conditions should be evident. The mesh and properties are shown in Figure 7. The deformed mesh (amplified by a factor of 50) is shown in Figure 8 for the time when the wave traveled half way up the block. The block had a release time which was

near the theoretical value. The stress distribution while qualitatively correct had some numerical "noise". This is probably due to a combination of factors including discretization error of the mesh, time integration errors (an implicit Newmark implementation was used), and lack of impact-release conditions. It is believed that the latter are small compared to the other errors; however, additional study is still necessary to confirm this impression.

4.4 Static and Dynamic Contact of a Sphere on a Rigid Surface

The last example considered is an elastic sphere contacting a rigid surface. The mesh and properties are shown in Figure 9. The results for three static loading conditions are shown in Figure 10. For the mesh considered these compare very favorably with previously published finite element results and the Hertz solution. It should be noted that the perturbed method gives accurate results for situations where only three segments are in contact. The dynamic results are also in agreement with previously published results although there is a difference between the finite element results and those of Hertz which are based upon a Ritz solution using the static mode. In this example the load-time curve is smooth, appearing similar to a sine-squared loading, and the impact-release conditions play a very minor role in defining even a theoretical solution.

5. FINDINGS AND RECOMMENDATIONS

The present study has considered the problem of dynamic contacts modeled by numerical methods. In particular an

augmented, perturbed Lagrangian formulation has been considered and is recommended as the best available method for inclusion in a design package to be developed by NCEL.

The augmented Lagrange multiplier method is an effective method of analysis for including contact capabilities in a finite element analysis program. It permits a very wide range of penalty parameters which effectively can be selected to avoid loss in numerical accuracy as well as avoiding ill-conditioning of the numerical formulation. Conventional penalty methods lose accuracy when the penalty parameter is too small and become ill-conditioned when the parameter is too large. As illustrated in even simple examples conventional penalty methods require use of a very carefully selected penalty parameter to attain viable solutions. For more complex problems we have experienced difficulties in selecting an appropriate value of the penalty parameter while still avoiding the above cited problems.

The above findings are based upon limited analyses; accordingly the following recommendations are made:

1. A three-dimensional implementation of the one- and two- pass augmented penalty method combined with a gap measured by the perturbed approach should be developed.
2. A detailed evaluation of the performance on dynamic problems involving friction and impact-release requirements of the type needed by NCEL should be made.
3. The penalty contact method should be evaluated in conjunction with models incorporating the non-linear constitutive models for concrete and geological materials.

4. General design goals for the class of problems to be considered should be constructed.
5. The complete problem methodology to be used in meeting the design goals identified in recommendation 4 should be tested on sample problems.

The development of a 3-dimensional implementation is necessary to assess possible complexities which may occur in the general setting. The generalization of the perturbed Lagrangian method to 3-dimensions is not trivial in specifying all possible segment configurations. Recently, it has become apparent that measuring the average gap for the two-pass method is nearly equivalent to the perturbed method. Accordingly, it is recommended to consider the development of the two-pass method, but with an average gap used, instead of individual gaps at nodes, to define the contact surface. An important aspect of using Newton's method is the proper derivation of a tangent matrix. Thus, efforts must be made to derive the consistent tangent array for all non-linear terms in the structural model - including the contact terms.

The behavior of dynamic contact problems analyzed without use of special contact-release conditions is promising. It has become apparent in our recent efforts in solving a variety of non-linear problems that care must be exercised in developing the algorithm for the step-by-step integration of the equations of motion. A methodology is now used in FEAP which has performed well for analyses which include large motions (e.g., analyses of free flight of structures representing flexible space-craft), as well as those which have non-linear material behavior. Our

analysis of the dynamic contact problem has also used this formulation. To date, we have used Newmark's method with the start condition at each time step the last converged "displacement" state. Initial velocities and accelerations are deduced from the Newmark equations to be consistent with this starting condition. We have also been extremely careful to include a proper linearization of specified boundary motions so that they are consistent with the algorithm for the active degrees of freedom. The implementation is then completed by computing iterative corrections to the nodal displacements and, subsequently, the nodal velocities and accelerations. Non-linear material models are integrated at each "stress point" using a local Newton iteration for the full increment of motion computed in the time step. We know that the implementation details for the Newmark method are sensitive in solving the contact problem. In using a previous implementation it was necessary to introduce contact-release conditions in order to obtain viable results [2]. Further analyses are required to ensure that accurate results may be obtained using the current implementation on a wide range of problems. In addition effort should be devoted to developing a formulation which includes a consistent treatment of friction in the formulation. Recent applications of operator split methods have been used to clearly explain the "radial return" algorithm for solving elasto-plasticity problems. The similarity between friction problems and elastic-plastic problems cannot be overlooked. It is recommended that an operator split formulation be pursued in an attempt to develop a consistent tangent

formulation for frictional contact problems. This formulation will need to be evaluated for dynamic problems to ensure that impact-release conditions are still not required.

The general design goals and loading states to be considered to achieve these goals needs to be clearly stated by NCEL to ensure that the additional developments are not required in developing a design package. This will involve some preliminary analyses to assess the levels of deformation and stress which can be anticipated in the facilities of interest to NCEL. It is particularly important to assess whether large strain effects or instability may result in any of the problems. This phase will identify the salient feature which the analysis program must include in order to develop a design oriented program system. Some attention needs to be devoted to the methods of showing results to the designer. For example, use of sensitivity analysis may provide important insight for the designer to effect changes in the system parameters and thus achieve the design objectives.

The above recommendations involve considerable effort and must be be given a priority ranking. For example, in establishing the design goals it will be necessary to specify computing requirements. This in turn depends upon the capabilities of particular computers available to NCEL and its contractors. This is currently a rapidly changing area and decisions must not be made too early in the setting of goals. On the otherhand, the assessment of the behavior of the augmented Lagrangian methcd with the other material models may proceed independent of which computer will ultimately be used.

Accordingly, while effort should be given early to all aspects of the problem those involving additional developments should be addressed first.

6. ACKNOWLEDGMENTS

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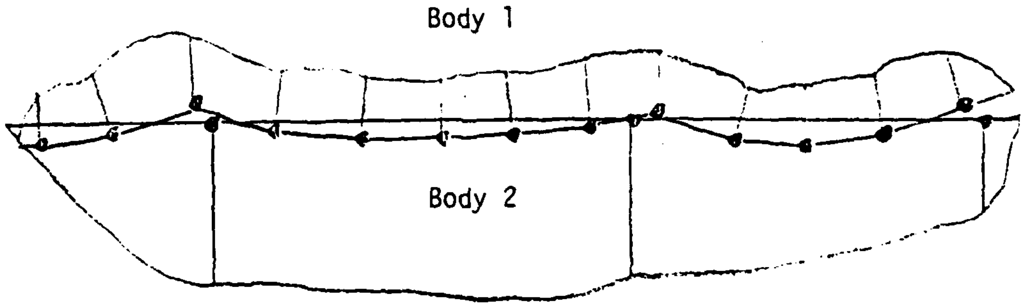


Figure 1. One-pass Lagrange Multiplier Results.
Different mesh spacing on two bodies.

Figure 2. Two Spring Problem
Displacements constrained to be equal

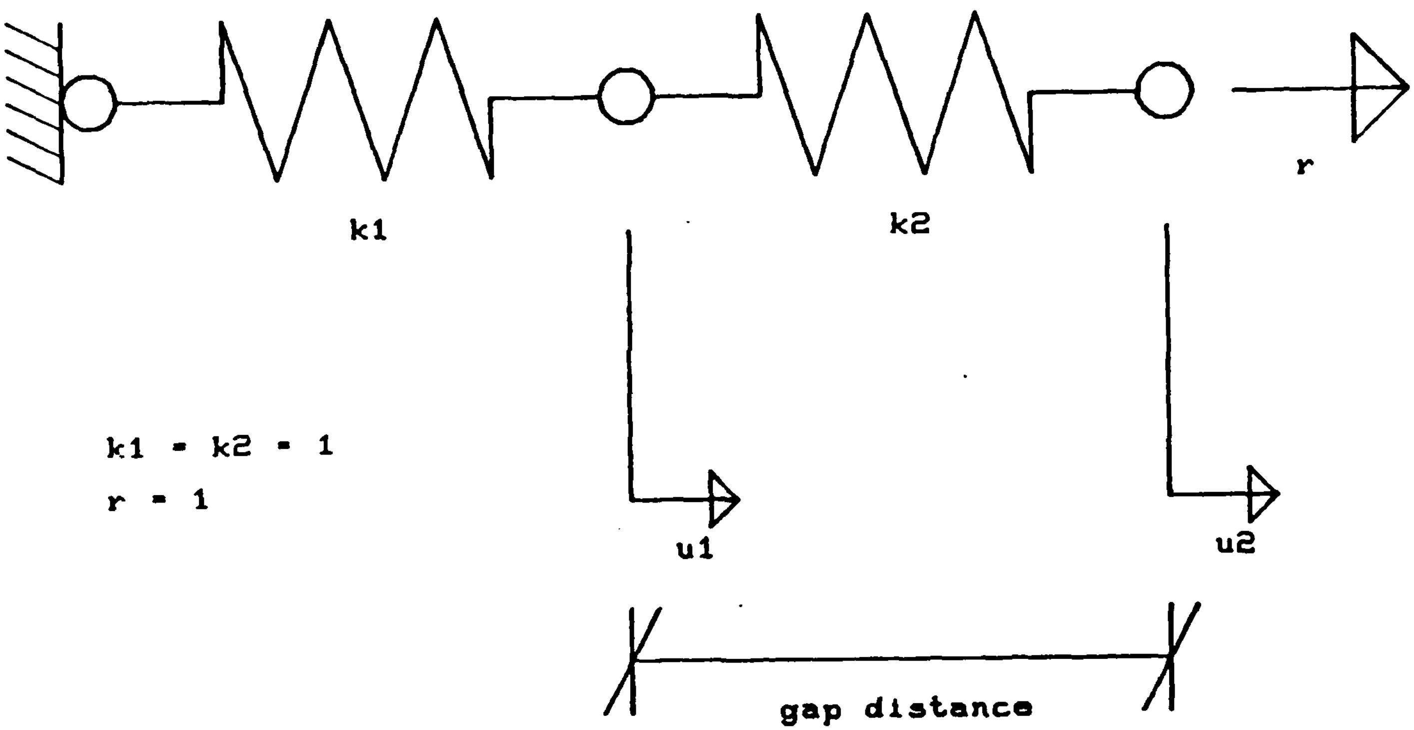


Figure 3.
Solution of the 2 Spring Problem
by the Augmented Lagrangian Method

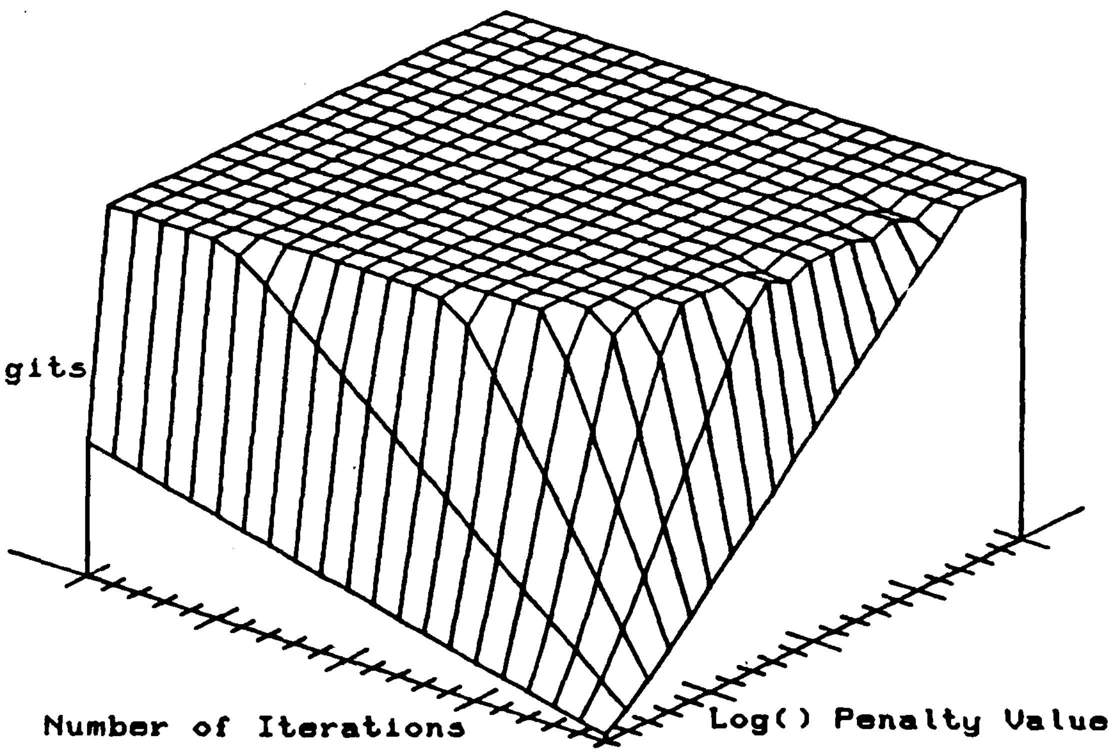


Figure 4. Rigid Punch into an Elastic Foundation: Finite Element Analysis using the Augmented Lagrangian Method

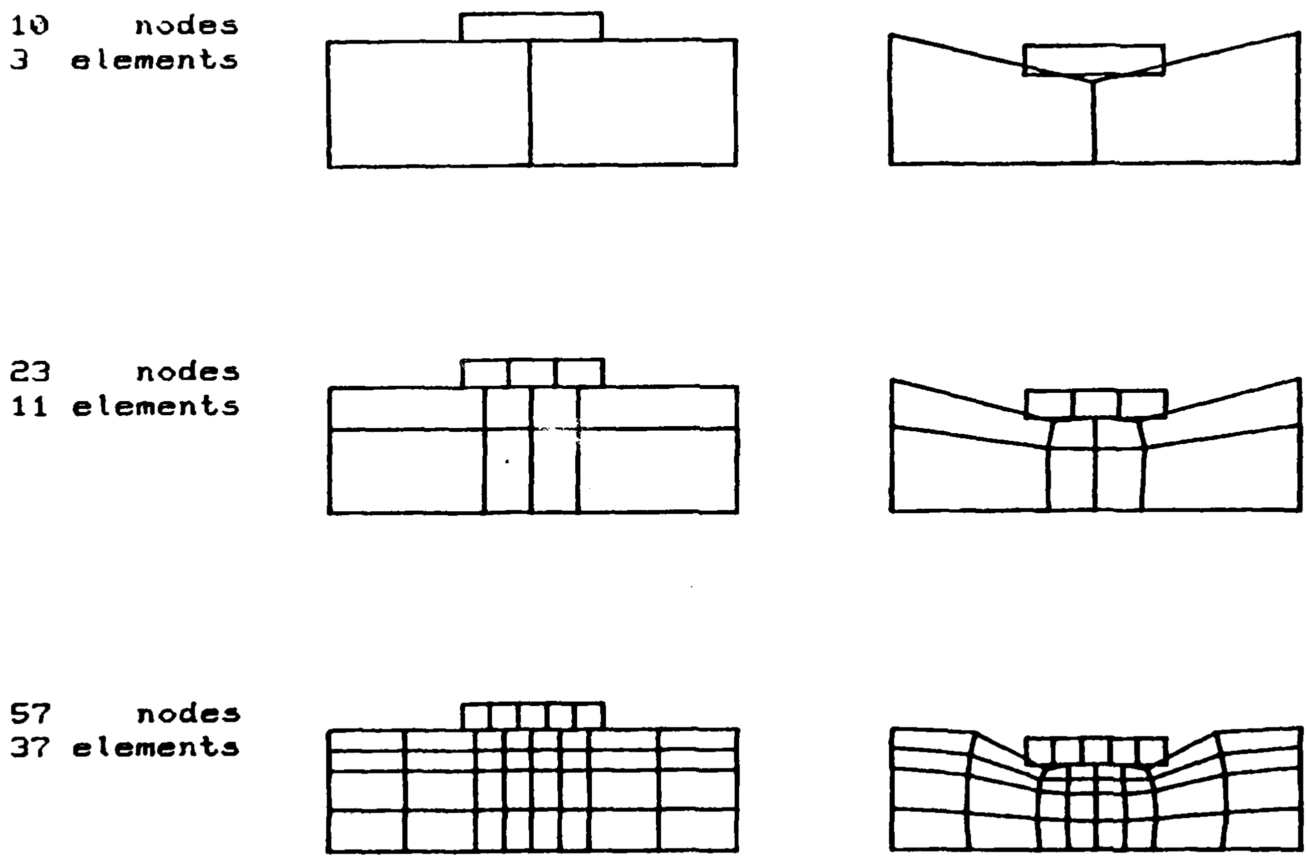


Figure 5. Results after 1 iteration for the 3 block problem

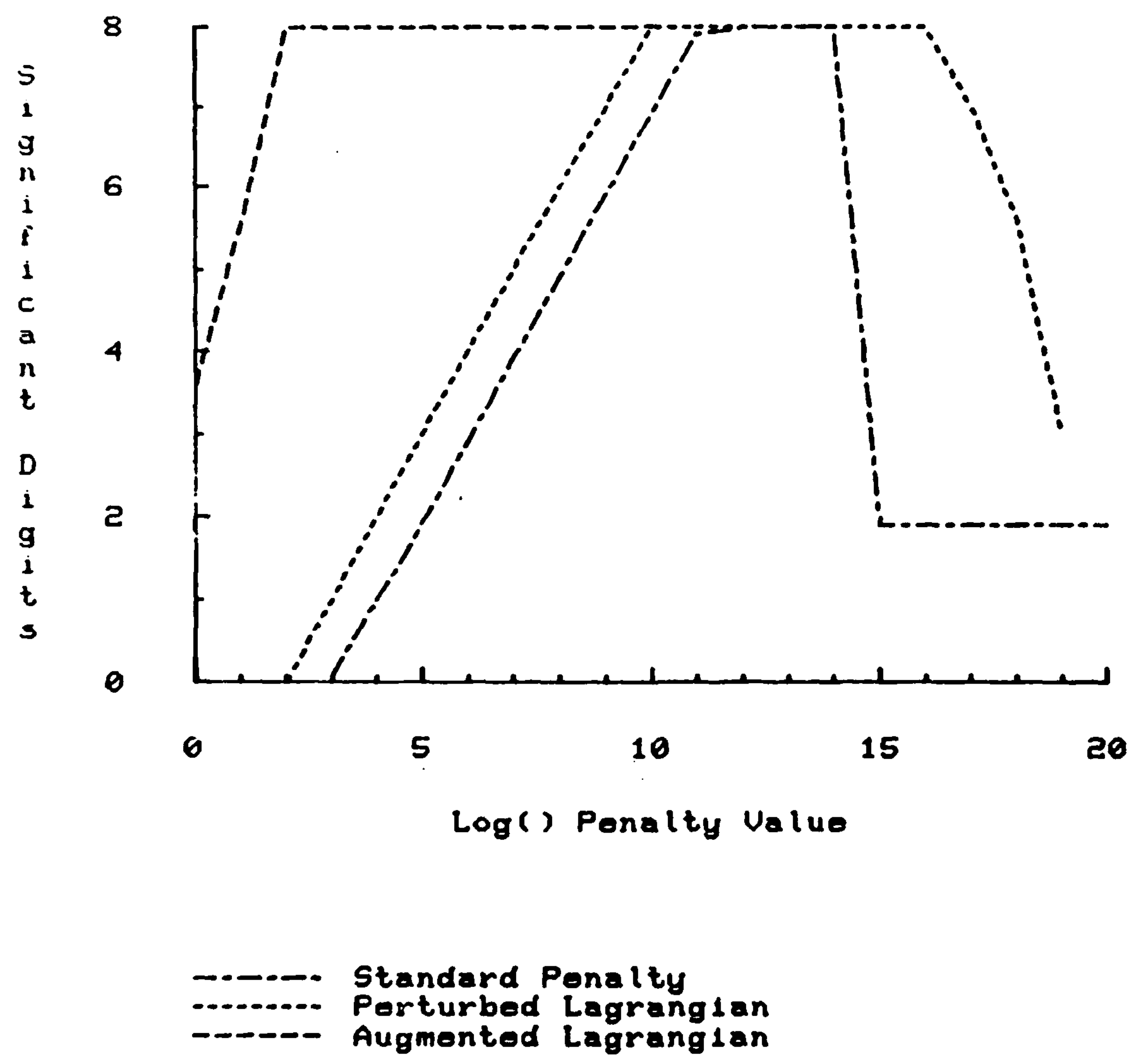


Figure 6. Comparison of Converged Results for the 3 block problem

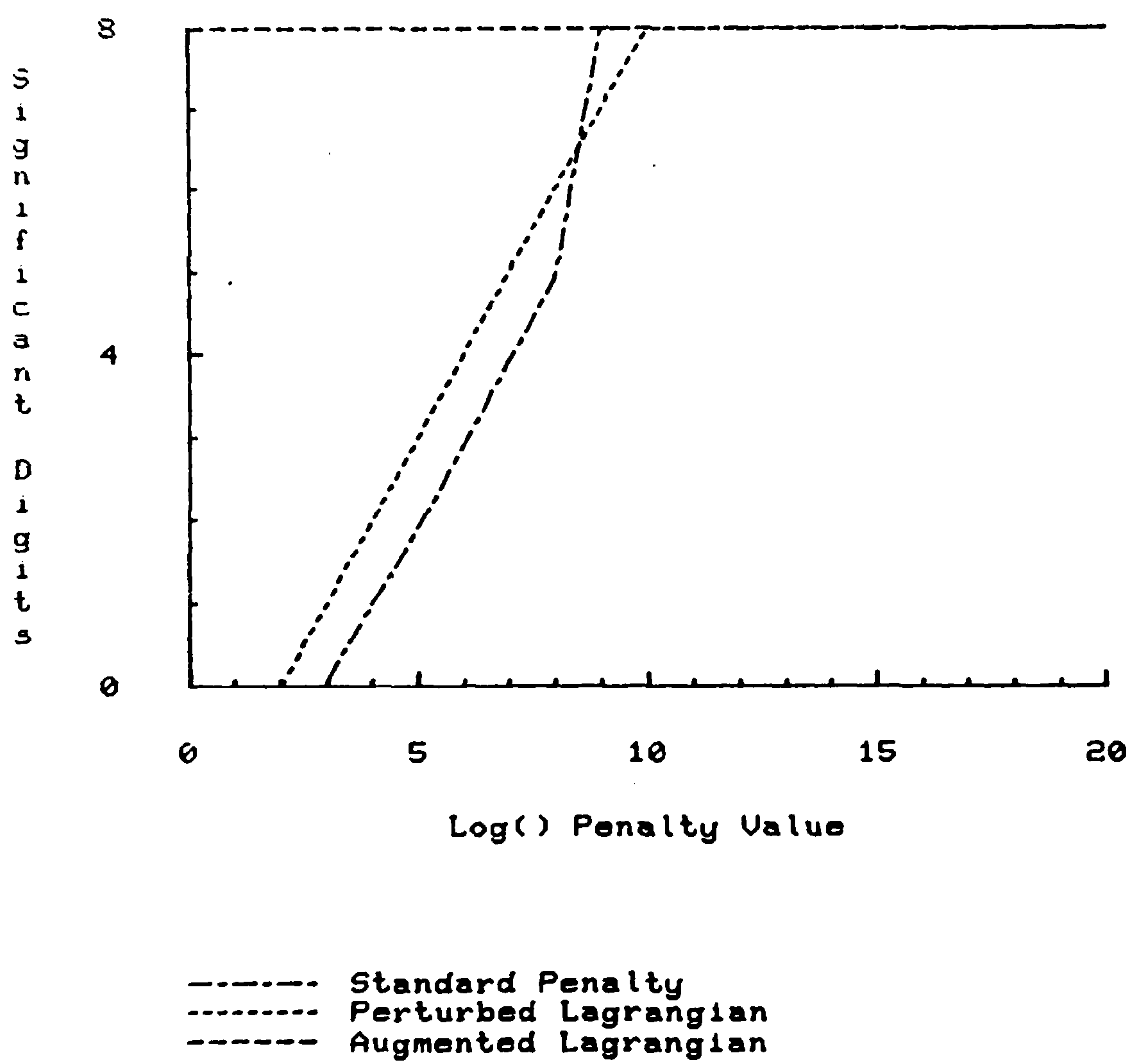
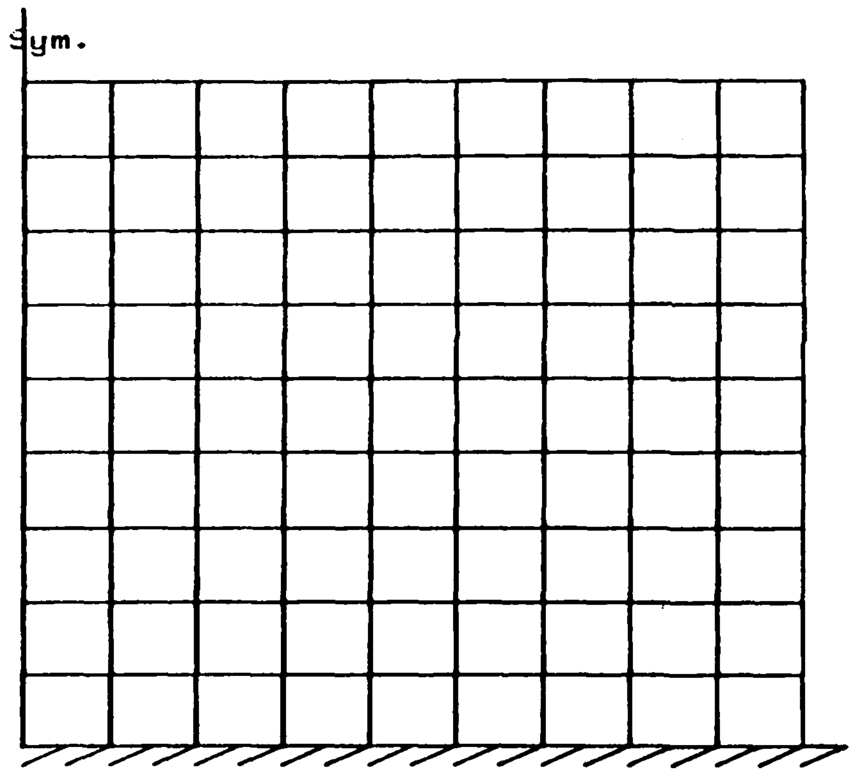
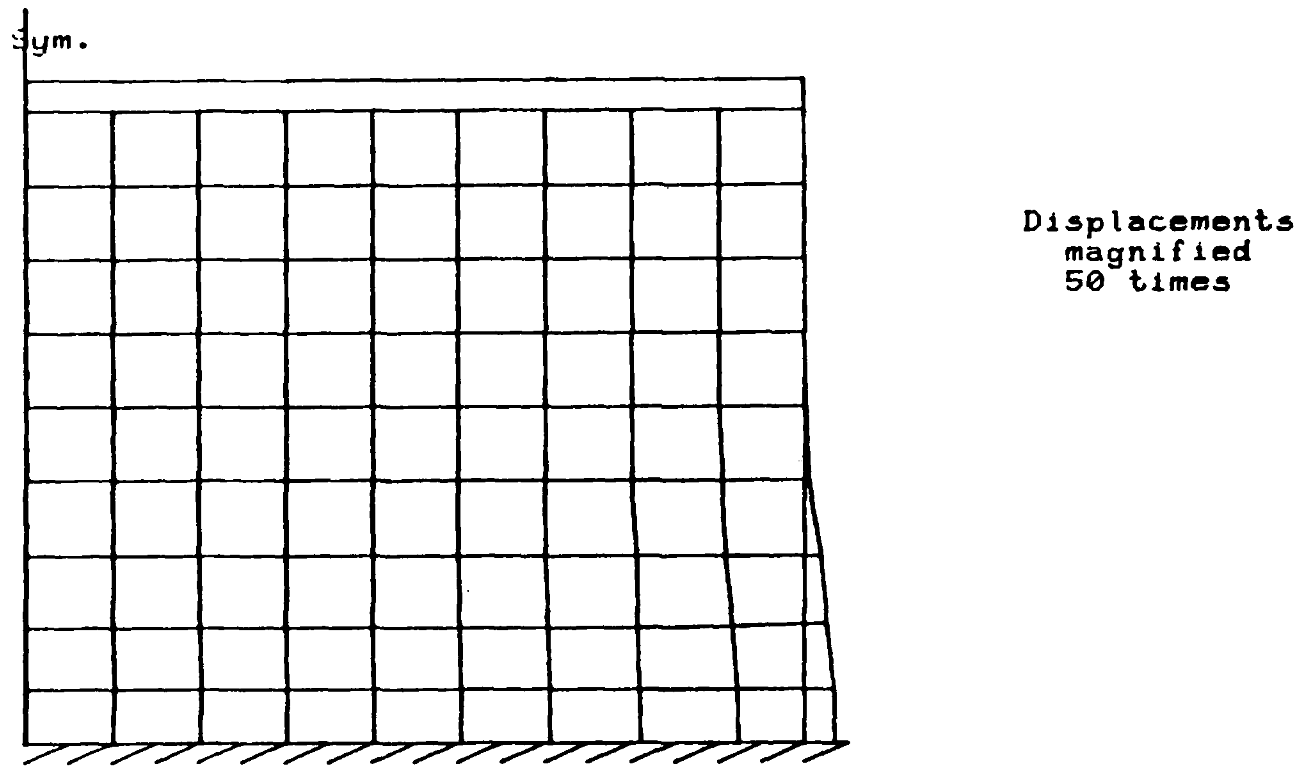


Figure 7. Finite Element Model of an Elastic Block in Dynamic Contact with a Rigid Surface



Young's modulus = 1000
Poisson's ratio = 0.30
Material density = 0.10
Initial velocity = 1.00
Length of sides = 9.00

Figure 8. Elastic Block in Dynamic Contact with a Rigid Surface (at time .0163)



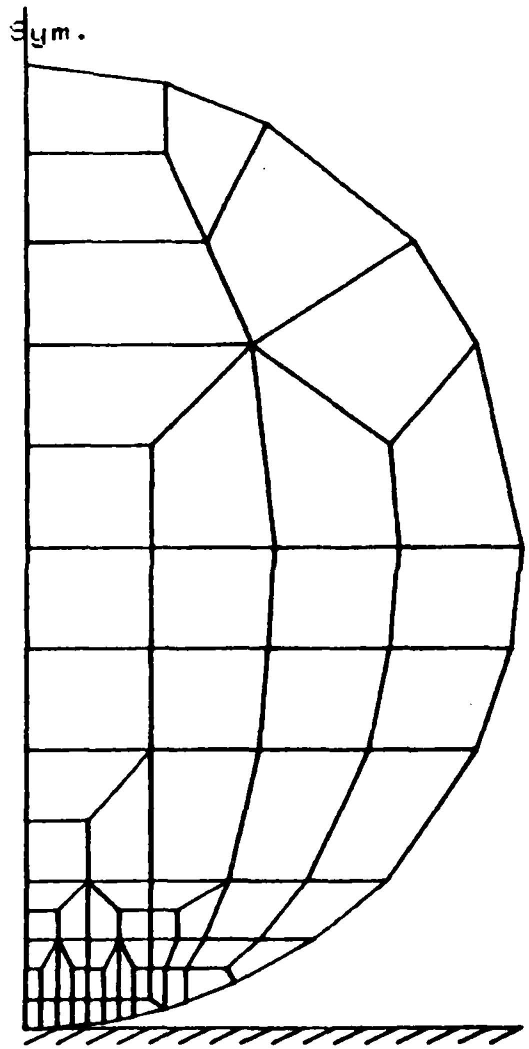


Figure 9.
Hertz Contact Problem

Young's modulus = 1000
Poisson's ratio = 0.30
Material density = 0.01
Radius of sphere = 8.00

Figure 10. Solution for the Hertz contact problem - static results

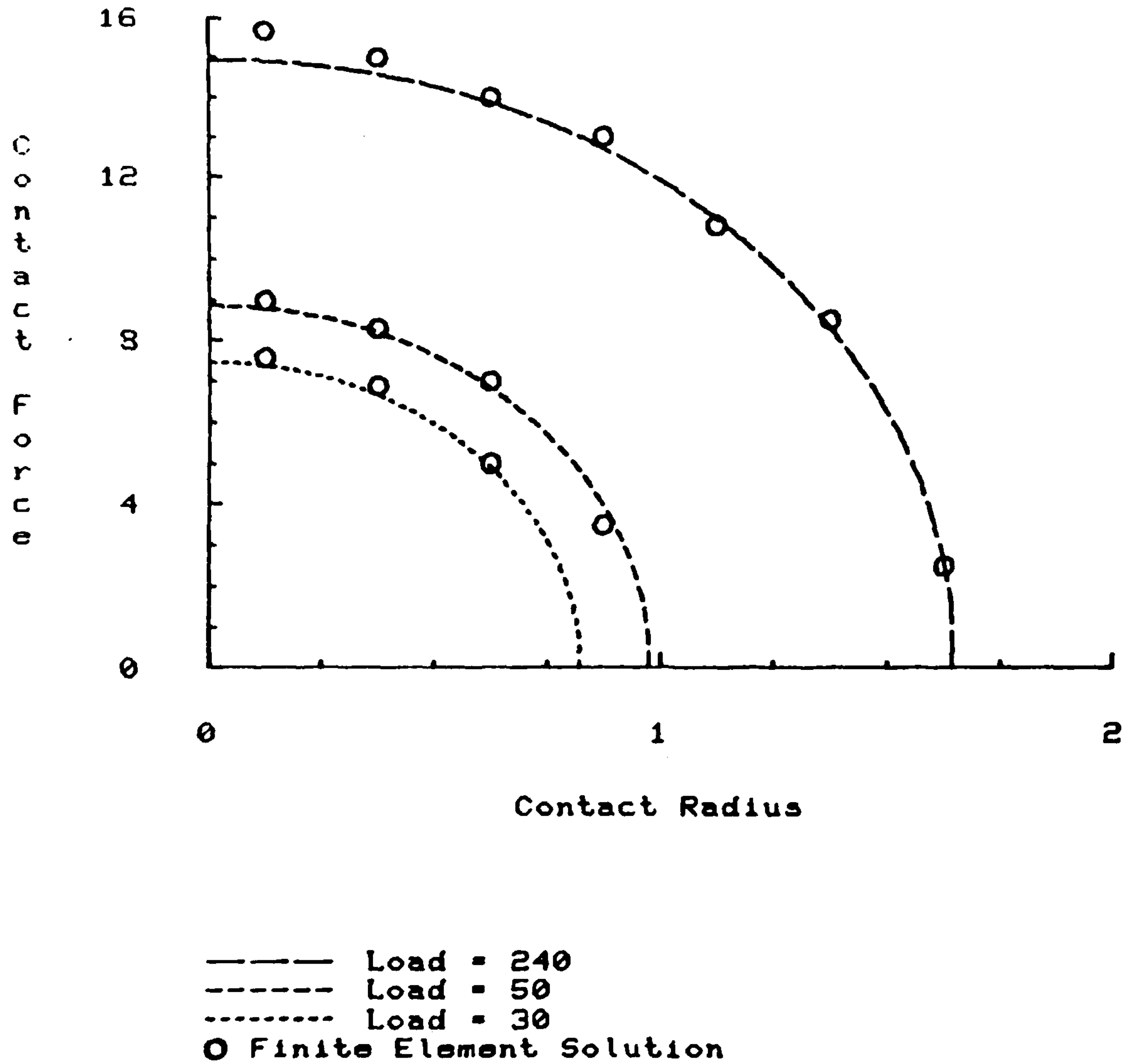
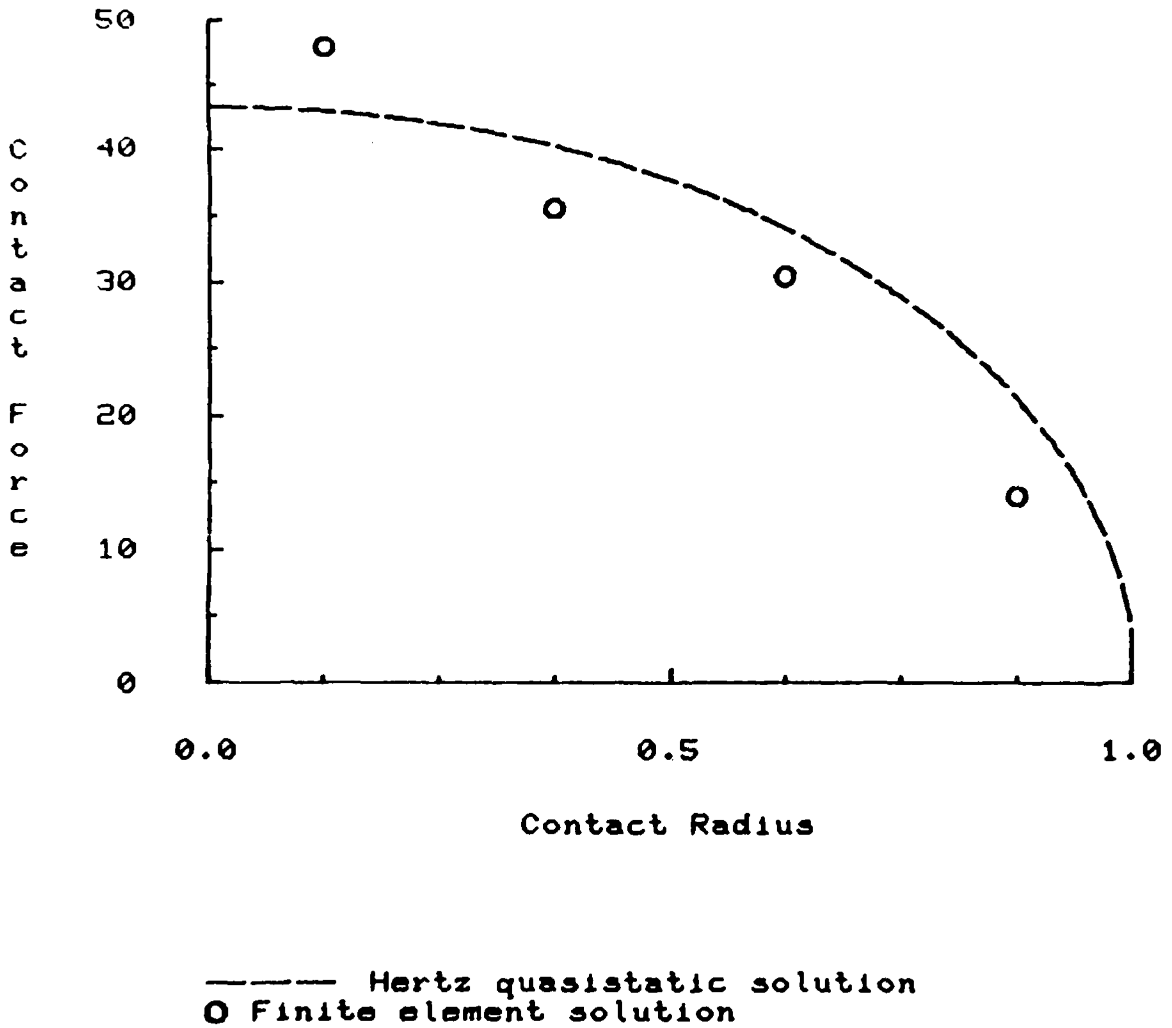


Figure 11. Dynamic contact of a sphere against a rigid wall



APPENDIX A. Contact Element for FEAP - Node-on-Node Case

```
      subroutine elmt12(d,ul,xl,ix,tl,s,p,ndf,ndm,nst,isw)
      implicit double precision (a-h,o-z)
c.... contact element for feap
      dimension d(1),xl(ndm,1),ul(ndf,1),ix(1),tl(1),s(nst,1),p(nst)
      dimension wd(2)
      common/cdata/o,head(20),numnp,numel,nummat,nen,neq,ipr
      common/eldata/dm,n,ma,mct,iel,nel
      save /cdata/,/eldata/
      data wd/4hslip,4hlock/
c.... transfer to correct processor
      go to (1,2,3,3,5,3,5,2,2,2), isw
c.... input the material properties
1     read(5,1000) idf,idir,ist,d(4),icelm,d(9)
      ist = max0(1,min0(2,ist))
      if(d(4).le.0.0d0) d(4) = 1.e-5
      write(6,2000) idf,idir,wd(ist),d(4),icelm,d(9)
      d(8) = icelm
      d(1) = idf
      d(2) = idir
      d(3) = ist
      return
c.... check element for errors
2     continue
      return
c.... compute the element tangent array
3     idf = d(1)
      idir= d(2)
      ist = d(3)
      gap = (ul(idf,3)-ul(idf,1)) + (xl(idir,3)-xl(idir,1))
      tt = ul(idf,2)
      e = 0.0
      if(gap.lt.d(4)) e = 1.0
      if(tt.lt.-d(4)) e = 0.0
      if(gap.lt.-d(4)) e = 1.0
      if(isw.ne.3) go to 4
      go to (30,31), ist
c.... sliding contact
30    i = ndf + idf
      s(i,idf) = e
      s(idf,i) = e
      s(i,i+ndf) = -e
      s(i+ndf,i) = -e
      s(i,i) = 1.0 - e
      return
c.... perfect stick contact
31    do 32 i = 1,ndf
      s(i,ndf+i) = e
      s(ndf+i,i) = e
      s(ndf+i,ndf+i) = 1. - e
      s(ndf+ndf+i,ndf+i) = -e
32    s(ndf+i,ndf+ndf+i) = -e
```

```
      return
c.... compute and output the element variables
4     continue
      if(isw.eq.6) go to 6
      write(6,2002) n,ma,tt,e
      return
c.... compute element mass arrays
5     continue
      return
c.... compute the element residual vector
6     p(idf+ndf) = gap
      go to (60,61), ist
60    p(idf) = -e*tt
      p(idf+ndf+ndf) = e*tt
      if(e.eq.0.0d0) p(idf+ndf) = -tt
      go to 64
61    do 62 i = 1,ndf
      p(i) = -e*ul(i,2)
      p(i+ndf+ndf) = e*ul(i,2)
62    if(e.eq.0.0d0) p(ndf+i) = -ul(i,2)
64    icelm = d(8)
      if(icelm.eq.n) p(idf+ndf+ndf) = p(idf+ndf+ndf) + d(9)
      return
c.... formats
1000  format(3i5,f10.0,i5,f10.0)
2000  format(5x,'contact element'/10x,'contact d.o.f. ',i5/10x,
     1          'coordinate direct.',i5/
     1       'contact condition ',a4/10x,'tolerance ',e15.5/
     2       'forced element',i6/'force value ',e13.5/)
2002  format('elmt',i5,'matl',i5,'force',e12.3,'e',f6.2)
      end
```

END

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